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Capacity Choice under Uncertainty with Product Differentiation

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1 Introduction

This article analyses a two-stage duopoly game, where firms set capacities in the first stage and prices in the second. Demand is stochastic when capacities are chosen and becomes known before the pricing stage starts. Capacity-then-price games with uncertainty are relevant for a great number of industries where capacity is costly and outputs cannot be stored, such as scheduled transport services, telecommunication services, electricity generation, hotels and so on. This article focuses on markets with exogenous horizontal and vertical product differentiation. The combination of uncertainty and product differentiation in capacity-then-price games has not been studied before. Prior literature addresses these two topics only separately.

The seminal article by Kreps and Scheinkman (1983) has spurred a wave of research into capacity-then-price games. Kreps and Scheinkman establish that such a two-stage game yields profit maximising capacities equal to profit maximising quantities in a one-stage Cournot game. Several authors (Hviid, 1991; Gabszewicz and Poddar, 1997; Reynolds and Wilson, 2000) study similar settings under demand uncertainty and find that equilibria in pure strategies either fail to exist, or only exist for specific configurations of the demand function. In contrast, De Frutos and Fabra (2011) do solve the capacity-then-price game and prove existence of the equilibrium by defining the capacity choice game submodular. Their findings are consistent with the more qualitative finding by Reynolds and Wilson (2000) that if an equilibrium exists, it is asymmetric in outcomes, despite a symmetric setting.

Benassy (1989) provides the key contribution in understanding the impact of substitutability in Bertrand-Edgeworth-Chamberlin models. He finds that a pure strategy equilibrium in prices does not exist for markets with close substitutes. He takes capacities as given, therefore his results may have implications only for the price subgame in models with endogenous capacities.

The study in this article adds to the literature by explicitly taking product differentiation into account. Although product differentiation may change the nature of capacity-then-price models, it has so far by-and-large been ignored in capacity-then-price games. For example, horizontal product differentiation can yield positive profits under Bertrand competition whereas vertical product differentiation would be a realistic source of asymmetries. To the best of our knowledge, Young (2010) provides the only attempt to model both. He claims that symmetric pure strategy equilibria exist for goods that are imperfect substitutes. Unfortunately, this finding appears to be based on a mistake, as we will argue in Section 3.

Our model allows for vertical and horizontal product differentiation to occur simultaneously in different degrees. We identify a minimum degree of vertical differentiation, relative to horizontal, for which the existence of subgame perfect Nash equilibria (SPNE) in pure strategies is guaranteed. To the best of our knowledge, this is the first study to find equilibria for capacity-then-price games under demand uncertainty without having to rely on mixed strategies and/or submodular games. Subsequently, we compare the capacity-then-price game with the standard Cournot model to analyse the effects of demand uncertainty and product differentiation on capacities and efficiency. Demand uncertainty results in higher (lower) market capacities and welfare efficiency if capacity costs are relatively low (high).

This study also relates to the literature on price dispersion over demand states. Dana (1999) finds that demand uncertainty and fixed costs may explain the existence of intrafirm price dispersion in competitive and oligopolistic markets. Our results show that Dana's finding that intrafirm price dispersion increases with the intensity of competition does not necessarily hold in the case of vertical

product differentiation. This finding provides an explanation for the mixed results in empirical studies, such as Borenstein and Rose (1994) and Gerardi and Shapiro (2009).

The remainder of this article is organised as follows. Section 2 outlines the model, followed by a discussion of the existence of equilibria in pure strategies in Section 3. Sections 4 and 5 analyse and discuss the model for those degrees of vertical differentiation that guarantee the existence of SPNE in pure strategies. The final section, Section 6, concludes.

2 The model

2.1 Basic set-up

Our model follows the framework used in Kreps and Scheinkman (1983). Two firms play a capacity-then-price game. Firms set capacities in the first stage and prices in the second stage. In both stages the firms act simultaneously. We add exogenous product differentiation to the model. Similar to Vives (1999, pages 144-147), we assume that the representative consumer has the following quadratic utility function:

$$U = \alpha(z_1 + \theta z_2) - (1/2)\beta(z_1^2 + z_2^2 + 2\phi z_1 z_2), \quad (1)$$

where z_i represents the firm specific output, α represents the reservation price, and β the direct demand sensitivity. We assume this demand sensitivity to be equal for both goods. The degree of horizontal product differentiation is captured by ϕ , which ranges from 0 for independent goods to 1 for

perfect substitutes.¹ Vertical product differentiation is represented by θ , where a unit value indicates that the goods are not differentiated. Without loss of generality, we appoint firm 1 as the high-quality firm by assuming $\theta < 1$.² Additionally, we assume that θ/ϕ is sufficiently large to ensure that firm 2 will install a positive capacity. For ease of exposition and interpretation, the parameters α and β are scaled in such a way that utility is measured in monetary terms.³

We will look at specific cases by imposing restrictions on the parameters in the utility function. First, setting $\phi < 1$ and $\theta = 1$ yields a model of horizontal product differentiation (with $\phi = 0$ providing the special case of independent goods) while setting $\phi = 1$ and $\theta < 1$ renders a model of vertical product differentiation. The limiting case of both models ($\phi = \theta = 1$) renders the case of a symmetric duopoly.

Utility is defined such that the marginal utility of output equals the marginal willingness to pay, allowing us to write inverse demand as:

$$\partial U / \partial z_1 = p_1 = \alpha - (z_1 + \phi z_2) / b, \quad \partial U / \partial z_2 = p_2 = \theta \alpha - (z_2 + \phi z_1) / b, \quad (2)$$

with $b = 1/\beta$. Both the reservation price, α , and the inverse of the direct demand sensitivity, b , determine the level of demand. Throughout this analysis, we focus on demand uncertainty in the level of demand through stochasticity in b , assuming that the reservation price is constant.

¹ Formally, $0 \leq \phi \leq 1$ ensures that the two goods are substitutes and U is strictly concave.

² Quality may also refer to perceived quality or desirability.

³ Alternatively, we may write Eq. (1) as a Lagrangian with a budget restriction and impose a unity value on the Lagrangian multiplier.

Firm profits are equal to revenue, that is p_i times z_i , minus costs. Firms maximise their profits by setting capacities, x_i , in the first stage and prices, p_i , in the second stage of the model. Output is constrained such that $z_i \leq x_i$. Firms face a cost of c_i per unit of capacity and are assumed to have no other costs. So, $C_i = c_i x_i$. We allow these costs to differ between firms. The only formal restriction on the cost parameter is that capacity costs are strictly positive for both firms. Apart from this formal restriction, one may argue that in practice the higher quality firm 1 is likely to have higher costs per unit of capacity. Examples of firms with higher (perceived) quality or desirability in combination with higher capital costs include legacy versus low-cost carriers, low- end versus high end hotels and renewable versus traditional electricity generation.

Kreps and Scheinkman (1983) establish that capacity-then-price games lead to Cournot outcomes, motivating the use of the single stage Cournot outcome of our model as a benchmark for further analysis. If demand is deterministic, the Cournot outcome becomes:

$$x_{1,c}^* = \bar{b}(2(\alpha - c_1) - \phi(\theta\alpha - c_2)) / (4 - \phi^2), \quad x_{2,c}^* = \bar{b}(2(\theta\alpha - c_2) - \phi(\alpha - c_1)) / (4 - \phi^2), \quad (3)$$

where \bar{b} is a positive parameter that will be discussed in more detail below. Without demand uncertainty, the capacities are equal to the outputs for both firms in the equilibrium: $z_i = x_i$. The model is asymmetric for $\theta < 1$ and equal capacity costs. In the Cournot benchmark, symmetry in profit maximising capacities is restored if the cost difference per unit of capacity equals the relative difference in reservation prices, $c_1 - c_2 = (1 - \theta)\alpha$. The level of demand and degree of horizontal product differentiation do not alter this condition.

2.2 Demand uncertainty

We now introduce demand uncertainty into the model. When setting capacities, firms know that different demand states will occur after capacities are chosen. Firms thus base their capacity decisions on a probability function of b , with support $(0, \alpha^{-1}]$ and mean \bar{b} .⁴ The expected profit for firm i equals:

$$\pi_i(x_1, x_2, b) = \int_0^{\alpha^{-1}} p_i(x_1, x_2, b) z_i(x_1, x_2, b) f(b) db - c_i x_i, \quad i = 1, 2. \quad (4)$$

Firms maximise expected profits by setting capacities in the first stage and prices in the second, with outputs following from capacities, prices and states of the world. The next section discusses how the states of the world, the level of demand and capacities are interrelated. The model needs to be solved by backward induction.

Apart from the profits of each firm, we look at the efficiency of the resulting equilibria over the different degrees of product differentiation, costs, and demand uncertainty. For the latter, we compare the efficiency of the standard Cournot outcome as shown in Eq. (3) and the results based on optimisation of Eq. (4). In line with earlier work, see e.g., Acemoglu et al. (2009), efficiency is defined as the ratio of welfare in equilibrium relative to the first-best outcome, with welfare defined as:

$$W(x_1, x_2, b) = \int_0^{\alpha^{-1}} U(x_1, x_2, b) f(b) db - c_1 x_1 - c_2 x_2, \quad (5)$$

⁴ Choosing the upper bound of α^{-1} scales the model and ensures that $D(0) \leq 1$.

where $U(x_1, x_2, b)$ is the utility function as specified in Eq. (1) after substituting equilibrium prices into outputs. Note that prices are a function of capacities and b : $p_i(x_1, x_2, b)$, hence $z_i(p_i) = z_i(x_1, x_2, b)$. To attain the first-best outcome the above welfare is maximised with respect to x_1 and x_2 given optimal prices and outputs for every level of demand b (see the discussion in Section 4 and Appendix D).

3 Existence of pure strategy equilibria

The existence of equilibria in pure strategies is not generally guaranteed in capacity-then-price games with uncertain demand. At the core of the problem lies the incentive in Bertrand-Edgeworth models for one firm to increase its price if the other firm's output is near capacity. As a result, the competitor's capacity constraint becomes binding, and the firm earns residual monopoly profits instead of Bertrand duopoly profits. Benassy (1989; p 227) establishes that for an equilibrium to exist, it should be impossible or irrational for any firm to saturate the capacity of its competitor(s) by raising its own price. In the case of deterministic demand, firms produce at full capacity by construction, so the problem does not arise and hence existence of a pure strategy equilibrium is guaranteed.

If demand is uncertain and capacity is costly, firms do not produce at full capacity for all levels of demand.⁵ Now, three situations may apply and hence three parameter regions can be distinguished. The boundaries for each region depend on the level of demand, b , relative to capacities, x_1 and x_2 . In the first region, demand is too low for any firm to produce at full capacity, and firms play unrestricted Bertrand in the price stage of the game. In the third region, demand is sufficiently high to have both firms produce at full capacity and charge clearing prices corresponding to their capacities, which boils down to the results reported by Kreps and Scheinkman (1983).

⁵ For low levels of demand, capacity utilization will be below 1, whereas for high levels of demand it may equal 1.

Based on the first and third region only, Young (2010) claims to have found an equilibrium in pure strategies, but this finding is based on ignoring the intermediate region, in which pure strategy equilibria in prices fail to exist (see e.g., De Frutos and Fabra (2011)).

The following example illustrates this point. Consider a Bertrand duopoly without product differentiation, and both firms producing near capacity. Since neither firm is restricted, both firms make zero profit in the Bertrand-Nash equilibrium. Either firm can however obtain a positive profit by setting a price above marginal costs, as this will saturate the other firm's capacity, implying that some consumers cannot switch to the other firm. For example, suppose firm 1 raises its price to saturate firm 2. This provides an incentive for firm 2, who is now capacity restricted, to raise its price as well. Actually, firm 2 may raise its price to just undercut firm 1's price without a loss in consumers. In turn, this provides an incentive for firm 1 to undercut firm 2's price. We then have the non-existence problem as established by Benassy (1989), which also holds for imperfect substitutes (Benassy, 1989, theorem 3).

We further explore the existence of pure strategy equilibria by distinguishing between two cases in our model. Appendix A proves that both firms will produce at any positive level of demand if and only if $\theta > \phi / (2 - \phi^2)$. We refer to this as 'mainly horizontal product differentiation', because the degree of vertical product differentiation is low (i.e. a high θ) relative to the degree of horizontal product differentiation. Both firms producing at any positive level of demand, results in the non-existence problem as discussed in the above example.

The non-existence of pure equilibria in case of 'mainly horizontal product differentiation' is in sharp contrast to our second case, which we refer to as 'mainly vertical product differentiation'. If

$\theta \leq \phi/(2-\phi^2)$, firm 2 does not want to produce as long as firm 1's capacity is not binding (see Appendix A). This implies that firm 2 does not have the option to saturate firm 1's capacity. Appendix B shows that for $\theta \leq \phi/(2-\phi^2)$ firm 1 does not have the incentive to saturate firm 2's capacity and attain residual monopoly profits over the residual demand instead of the contestable monopoly profits. Therefore, we conclude that the existence of SPNE in pure strategies is guaranteed for $\theta \leq \phi/(2-\phi^2)$.

The remainder of this article focuses on the 'mainly vertical product differentiation' case, for which the existence of SPNE in pure strategies is guaranteed. Such a setting would be relevant in real life for markets like air transport (low-cost carriers versus legacy carriers), hotels (low- end versus high-end hotels), and similar markets. Throughout the subsequent analysis, the degree of vertical product differentiation is bounded by a lower bound $\underline{\theta}$ guaranteeing the capacity of the low-quality firm to be positive, and by an upper bound $\bar{\theta} = \phi/(2-\phi^2)$ as determined in Appendix A guaranteeing the existence of an SPNE in pure strategies.

4 Mainly vertical product differentiation

As in the previous section, three parameter regions are distinguished. The boundaries for each region – contested monopoly, residual monopoly and Bertrand Edgeworth duopoly – depend on the level of demand, b , relative to capacities, x_1 and x_2 .

4.1 Contested monopoly region

If demand is sufficiently low, firm 2 will not produce, allowing firm 1 to set its price above the Bertrand level. However, firm 1 has to take into account the possibility that firm 2 may produce. Therefore, firm 1

offers output at the limit price, where firm 2 will just not produce. This limit price can be found by setting firm 2's best response in prices to zero and solve for p_1 :

$$p_{1,CM}^* = \alpha(\phi - \theta) / \phi. \quad (6)$$

The price is positively related to the reservation price and to the level of vertical product differentiation. A smaller quality difference (i.e a higher value for θ) leaves less room for the monopolist to reap quality rents, which is what one would expect from a limit price. Substituting $p_{1,CM}^*$ and $z_2 = 0$ into firm 1's demand function gives the equilibrium output:

$$z_{1,CM}^*(p_{1,CM}^*) = \alpha b \theta / \phi. \quad (7)$$

Both the equilibrium price and output are increasing in the degree of horizontal differentiation, implying that firm 1's profits are higher if the outputs are more horizontally differentiated. The equilibrium price increases and the output decreases in the degree of vertical product differentiation. For $\theta \geq 1/2$, firm 1's equilibrium profit increases in the degree of vertical differentiation, i.e. decreases in θ . The upper bound of the first region can be found by equating Eq. (7) to firm 1's capacity and solve for b :

$$b_{CM} = x_1 \phi / \alpha \theta. \quad (8)$$

Appendix C shows that if $\theta \leq \phi/2$, a special case arises. In this case, firm 2 cannot produce profitably whilst setting a positive price for its output. As a result, the contested monopoly becomes a pure monopoly in this first region. Finding the equilibrium price and output for firm 1 and the accompanying

boundary only requires substituting $\theta = \phi/2$ into Eq. (6), Eq. (7), and Eq. (8). Whether the first region is a contested or pure monopoly region, does not affect the subsequent analysis for regions two and three.

4.2 Residual monopoly region

If the level of demand exceeds the level determined in Eq. (8), firm 1 produces at full capacity, whereas firm 2 serves residual demand as a monopolist. Substituting $z_1 = x_1$ into the inverse demand function and rewriting for z_2 yields the demand function for output 2:

$$D_{2,RM}(p_{2,RM}, x_1) = \alpha b \theta - b p_{2,RM} - x_1 \phi. \quad (9)$$

The profit for firm 2 simply equals price times demand minus capacity costs. The profit maximising price for firm 2 now follows directly from rewriting the first order condition:

$$p_{2,RM}^* = (\alpha b \theta - x_1 \phi) / 2b. \quad (10)$$

The equilibrium price of firm 2 is increasing in the firm's own quality and decreasing in the capacity of firm 1, the latter clearly reflecting the nature of firm 2 having a monopoly over residual demand. Substituting $p_{2,RM}^*$ into the demand function for output 2 yields the equilibrium output:

$$z_{2,RM}^* = (\alpha b \theta - x_1 \phi) / 2. \quad (11)$$

Both the equilibrium price and output for firm 2 are increasing in the level of demand b , and decreasing in firm 1's capacity. Vertical product differentiation decreases firm 2's profits because it gives firm 1 more possibilities to exercise its contestable monopoly power in the first region.

Since firm 1 is capacity restricted in the second region, its optimal price is the clearing price, given firm 2's optimal behaviour. Substituting $z_1 = x_1$ and $z_{2,RM}^*$ into the inverse demand function of output 1, Eq. (2), gives the clearing price:

$$p_{1,RM}^* = (\alpha b(2 - \theta\phi) - x_1(2 - \phi^2)) / 2b. \quad (12)$$

The equilibrium price of firm 1 is positively related to the level of vertical product differentiation (or the quality difference) and negatively to its own capacity, which is a common finding in capacity restricted price games. The upper bound of the second region can be found by equating $z_{2,RM}^*$ to firm 2's capacity and solve for b :

$$b_{RM} = (x_1\phi + 2x_2) / \alpha\theta. \quad (13)$$

4.3 Bertrand Edgeworth duopoly region

In the third region, both firms are capacity restricted, i.e. the outputs associated to their optimal prices exceed their capacities. Both firms therefore produce at full capacity and set clearing prices to maximise profits:

$$p_{1,BE}^* = \alpha - (x_1 + x_2\phi) / b, \quad p_{2,BE}^* = \alpha\theta - (x_1\phi + x_2) / b. \quad (14)$$

Both prices are negatively related to the firms' joint capacity (corrected for horizontal product differentiation), which is a common feature of Bertrand Edgeworth models. Having expressed all equilibrium prices and outputs in terms of capacity and the level of demand, b , we can now solve the capacity stage.

4.4 Capacity stage

In the capacity stage, we substitute all equilibrium outcomes, prices and outputs, into the profit function as defined in Eq. (4):

$$\pi_1(x_1, x_2, b) = \int_0^{b_{CM}} p_{1,CM}^* z_{1,CM}^*(b) f(b) db + \int_{b_{CM}}^{b_{RM}} x_1 p_{1,RM}^*(x_1, b) f(b) db + \int_{b_{RM}}^{\alpha^{-1}} x_1 p_{1,BE}^*(x_1, x_2, b) f(b) db - c_1 x_1, \quad (15)$$

and for firm 2:

$$\pi_2(x_1, x_2, b) = \int_{b_{CM}}^{b_{RM}} p_{2,RM}^*(x_1, b) z_{2,RM}^*(b, x_1) f(b) db + \int_{b_{RM}}^{\alpha^{-1}} p_{2,BE}^*(x_1, x_2, b) x_2 f(b) db - c_2 x_2. \quad (16)$$

The three separate integrals in Eq. (15) reflect the relevant three regions - contested monopoly, residual monopoly and Bertrand Edgeworth duopoly - for firm 1, whereas the two integrals in Eq. (16) reflect the ones relevant for firm 2. Note that the domain of integration for each region depends on these capacities. Both firms maximise their expected profits by setting capacities simultaneously.

Solving this maximisation problem, and thereby the model, requires information on the distribution of b .

Here we assume that b has a uniform distribution over the interval $(0, \alpha^{-1}]$, where the upper bound

scales the model such that $D(0) \leq 1$. For this uniform distribution, the first order conditions for profit maximisation for both firms are:

$$\begin{aligned} \frac{\partial \pi_1(x_1, x_2, b)}{\partial x_1} = & -\frac{\alpha(x_1(\phi + \theta) - \theta)}{\theta} - \frac{\alpha x_2(4x_1x_2 + \phi(2x_1^2 + x_2^2))}{(x_1\phi + x_2)^2} \\ & + \frac{2\alpha}{x_1\phi + 2x_2 + \theta} \left(x_1(x_2(2 + \phi^2) + 2x_1\phi - \theta) + \frac{(2x_2 + x_1\phi - \theta)(x_1 + x_2\phi)(2x_2 + \theta)}{x_1\phi + 2x_2 + \theta} \right) - c_1 \stackrel{!}{=} 0, \end{aligned} \quad (17)$$

and

$$\frac{\partial \pi(x_1, x_2, b)}{\partial x_2} = \frac{\alpha x_2^2(3x_1\phi + 2x_2)}{2(x_1\phi + x_2)^2} + \frac{\alpha(x_1\phi - \theta)(4x_2(x_2 + x_1\phi + \theta) - \theta^2 + x_1^2\phi^2)}{(2x_2 + x_1\phi + \theta)^2} - c_2 \stackrel{!}{=} 0. \quad (18)$$

Solving the above first order conditions for capacities, results in analytical intractable expressions for the best response functions. We therefore continue by presenting numerical rather than analytical results.

Before turning to the numerical results in the next section, we first characterise the normative first-best solution. The first-best solution both includes optimal capacity levels per firm and optimal output levels given capacity. The same two-stage approach as applied above can be used to derive the first-best solution. In general, for ‘mainly vertical product differentiation’, in the social optimum only the (perceived) qualitative superior output, in our case firm 1’s output, should be produced until its capacity is restricted. Only if firm 1’s capacity is restricted, welfare increases by producing output 2. This optimal output scheme has, like the market scheme, three regions. In the first region, only firm 1 produces. In the second region, firm 1 produces at its capacity and firm 2 produces. In the last region both firms produce at their capacity. The single exception is when capacity costs of firm 1 are lower or equal to firm

2's costs, in that case welfare is always maximised by supplying only the (perceived) qualitative superior output and only two regions, firm 1 restricted or not restricted, are applicable.

Appendix D shows that the resulting optimal welfare for $\theta \leq \phi / (2 - \phi^2)$, and b uniformly distributed over the interval $(0, \alpha^{-1}]$, equals:

$$\begin{aligned} W_{fb} &= x_1^2 / 4 + \alpha x_1 (\alpha - x_1) / (\alpha + x_1) - c_{x,1} x_1 & \forall c_{x,1} \leq c_{x,2}, \\ W_{fb} &= x_1^2 / 4 + \Theta (x_1 (\phi - \theta) + x_2) / \alpha \theta + (1 - (\phi x_1 + x_2) / \alpha \theta) \Omega - c_{x,1} x_1 - c_{x,2} x_2 & \forall c_{x,1} > c_{x,2}. \end{aligned} \quad (19)$$

with $\Theta = \alpha (x_1^2 (4\phi + \theta^3 + \theta\phi^2 - 2\theta^2\phi) + x_1 x_2 (2\theta(\theta - \phi) + 4) + \theta x_2^2) / 4 (x_1 (\theta + \phi) + x_2)$,

and $\Omega = \alpha (x_1 (x_1 (\phi - \theta) + (\alpha\theta + (1 - \theta\phi)x_2)) + x_2 \alpha \theta^2) / (\alpha\theta + x_1 \phi + x_2)$.

Taking the first order conditions with respect to capacities of the welfare function defined in Eq. (19) and solving for capacities yields the first-best solution. Like the market outcome, the solution in first-best outputs is analytically intractable.

5 Results

Numerical analysis is required to solve the model as presented in the previous section. In discussing the results we focus first on resulting capacity levels, then on efficiency and finally on price dispersion. In each discussion we analyse the effect of demand uncertainty, the degree of vertical product differentiation and asymmetry in costs per unit of capacity.

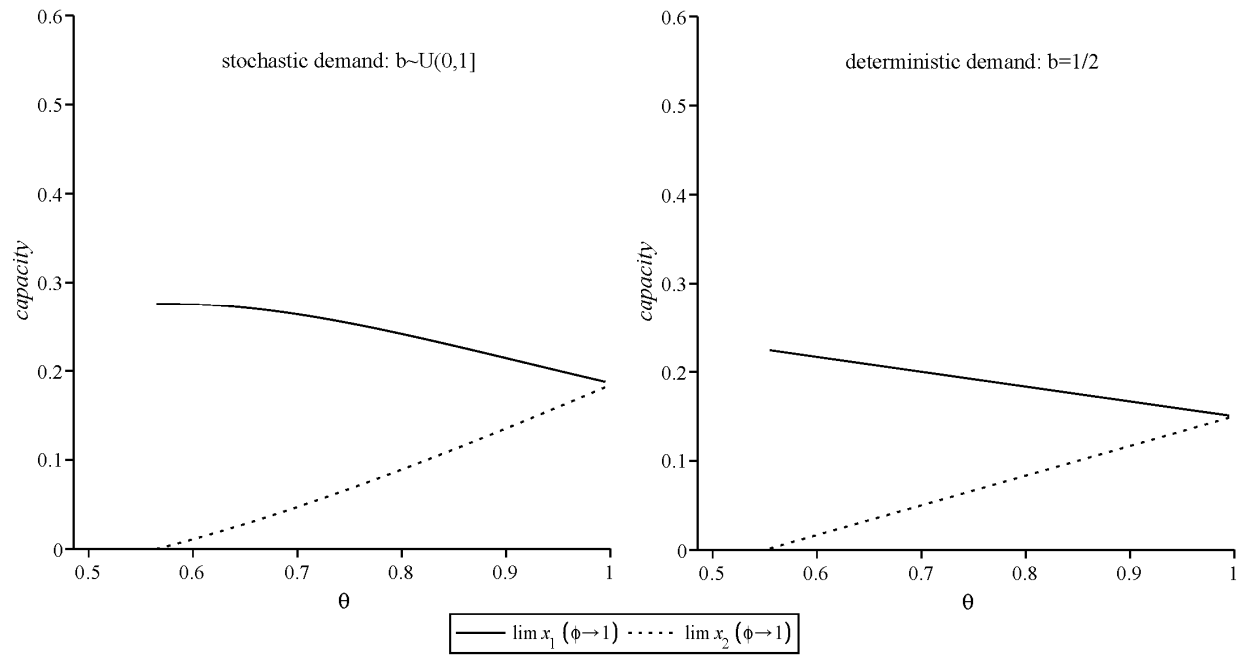
The model allows us to present numerical results over the entire relevant domain without making arbitrary assumptions on parameter values other than on the cost parameters. Since the upper bound for b is α^{-1} , setting α to unity scales all prices, outputs and quantities and boundaries of regions as fractions of the reservation price and maximum demand respectively.⁶ Parameters ϕ and θ are bounded by the restrictions that guarantee existence of a duopoly SPNE in pure strategies as discussed in Sections 2 and 3. We vary the two parameters within these boundaries to assess their impact. Since the degree of horizontal product differentiation, ϕ , only has a minor effect on any of the outcomes presented here, only results for the limiting case of its value nearing unity are reported. We distinguish between cases where firms have equal costs and cases where firm 1 has higher costs per unit of capacity.

5.1 Profit maximising capacities

Figure 1 plots the profit maximising capacities of both firms for different values of θ at equal cost levels ($c_1 = c_2 = 0.1$) for both firms. The left panel of Figure 1 shows these outcomes for uncertain demand with b uniformly distributed over the interval $(0,1]$. The right panel shows deterministic demand with $\bar{b} = 1/2$ based on the Cournot benchmark as expressed in Eq. (3).

⁶ We emphasize again that setting the upper bound for b to α^{-1} is not an arbitrary assumption, but merely scales the model.

Figure 1 Profit maximising capacity as a function of θ ($c_1=c_2=0.1$).



The patterns in capacity levels are very similar for both cases. The patterns remain similar when varying the cost level. Whether firms install higher capacities under uncertainty, depends exclusively on the cost level relative to the reservation price. For $c_1 = c_2 = 0.1$, Figure 1 clearly shows that stochastic demand, with the same expected level of demand, leads to higher capacities compared to deterministic demand for all degrees of product differentiation. This implies that given this cost level and the reservation price, it is rational for both firms to have spare capacity for low levels of demand b , rather than being capacity constrained for higher levels of demand, whereas for higher cost levels the reverse holds.

We explore the impact of costs on capacity by looking at the case where costs differ between the firms. In the Cournot benchmark, symmetry in profit maximising capacities is restored by imposing a cost difference, such that $c_1 - c_2 = (1 - \theta)\alpha$. Figure 2 plots the profit maximising capacities of both firms for

different values of θ given different cost levels for both firms ($c_1=0.4; c_2=0.1$). In the Cournot benchmark (right panel), this would imply a symmetric outcome in capacities if $\theta=0.7$.

Figure 2 Profit maximising capacity as a function of θ ($c_1=0.4, c_2=0.1$).

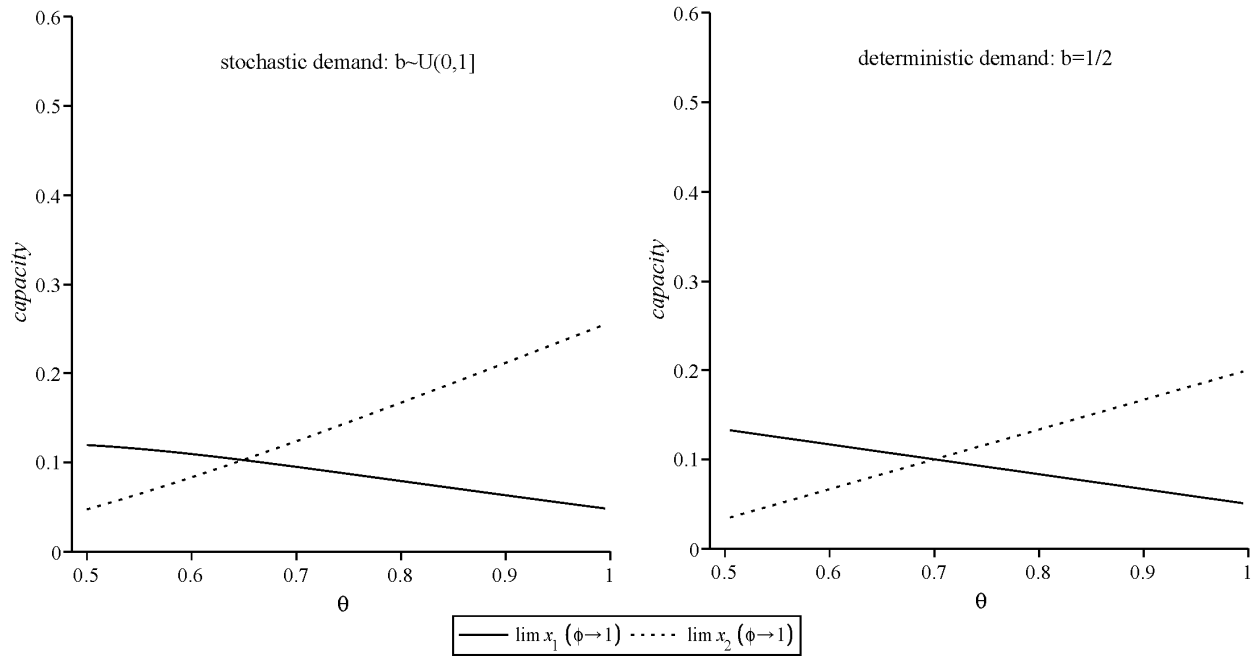


Figure 2 shows that symmetry in capacities (i.e. the intersection of the curves) under uncertain demand occurs at a higher degree of vertical product differentiation (i.e. lower θ) than in the Cournot benchmark. Stated alternatively; for those parameters that yield symmetric outcomes in the Cournot benchmark, our findings show that firm 2, the low-cost, low-quality firm, provides more capacity than firm 1 if demand is stochastic. The finding of an asymmetric result for input values that would in the deterministic case yield symmetric outcomes is consistent with earlier results, reported by Reynolds and Wilson (2000) and De Frutos and Fabra (2011). Firm 1's high capacity costs withhold it from serving

occasional high levels of demand, leaving more room for firm 2 to serve the market than it would have in the deterministic case.⁷

5.2 Welfare maximising capacities

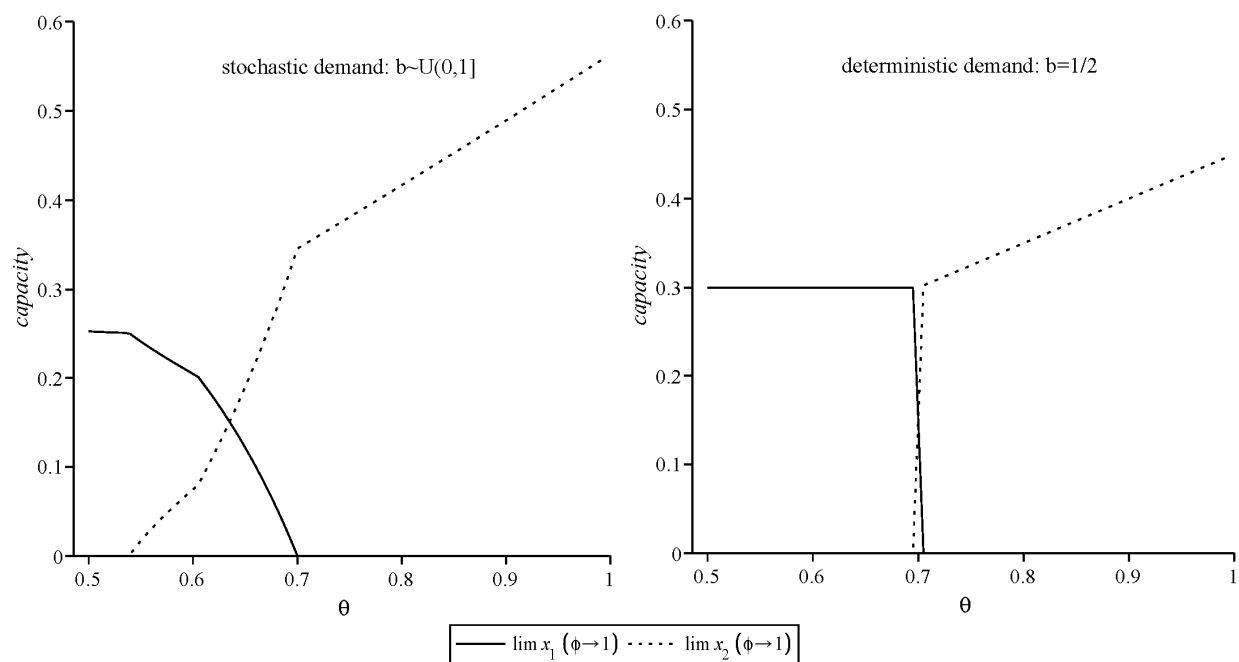
Welfare maximisation requires that only firm 1 produces for any (perceived) quality difference and equal costs, and both for stochastic and deterministic demand. For $c_1 = c_2 = 0.1$, the welfare maximising capacity is higher under uncertainty. The difference between welfare maximising capacities under stochastic and deterministic demand essentially depends on the trade-off between two factors. First, if demand exceeds capacity, welfare is lost due to demand not being served. Second, if demand is below capacity, costs of excess capacity are not met. If the overall cost level is low, the relative importance of the second effect is smaller, and hence the welfare maximising capacity is higher. If the cost level is higher, welfare maximising capacities are lower in the stochastic case than in the deterministic case.

The welfare maximising capacities in case of different capacity costs are shown in Figure 3. For deterministic demand, the welfare maximising situation is that either the cheapest or the highest quality firm produces (Figure 3, right panel). If the lower cost firm produces, its optimal capacity is increasing in θ , which follows directly from the utility function in Eq. (1). For stochastic demand, the transition is more gradual, with a wide range of values for θ where both firms have positive capacities. Again, the trade-off between foregone welfare due to excess demand and costs of excess capacity lie at the heart of the difference. If the difference in quality levels is high, high quality capacity would be preferred in general, but it would be inefficient to build sufficient capacity to serve peak demand. It could however be efficient to have the high-quality firm serve ‘normal’ demand levels and build additional ‘cheap’

⁷ If the cost level for both firms increases equally, the difference between restoring symmetric outcomes for stochastic versus deterministic demand, measured in θ , becomes smaller. For high cost levels, symmetry is restored at $\theta = 1 - (c_1 - c_2) / \alpha$ for both stochastic and deterministic demand.

capacity to serve peak demand. Once the degree of θ reaches the threshold where $c_1 - c_2 = (1 - \theta)\alpha$, it is no longer efficient to have the high-quality firm producing, as its quality advantage does not outweigh its cost disadvantage.

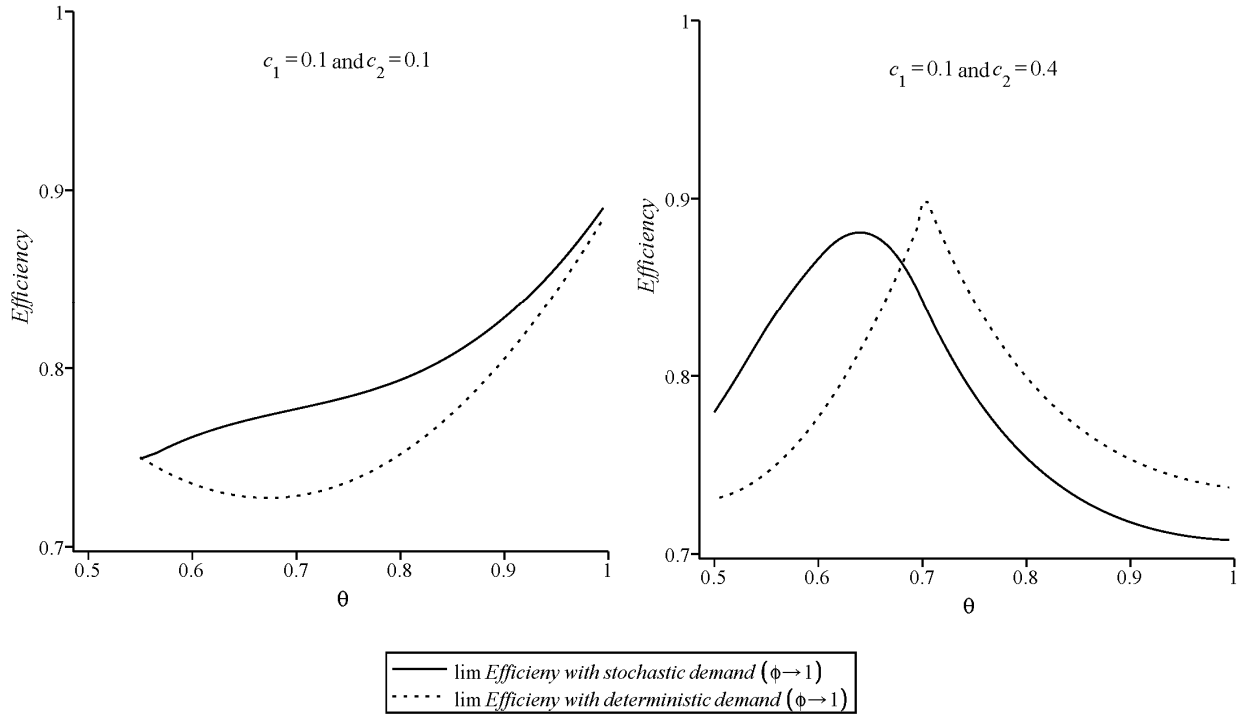
Figure 3 Welfare maximising capacity as a function of θ ($c_1=0.4, c_2=0.1$).



5.3 Efficiency

Based upon the profit- and welfare maximising capacities, we construct the measure for efficiency as discussed in Section 2 and compare this measure between stochastic and deterministic demand over the valid degrees of vertical product differentiation. Figure 4 graphs the efficiency for the stochastic (solid lines) and deterministic (dashed lines) cases for equal (left) and unequal (right) costs per unit of capacity.

Figure 4 Efficiency as a function of θ (left panel: $c_1=0.1, c_2=0.1$, right panel: $c_1=0.4, c_2=0.1$).



For $c_1 = c_2 = 0.1$, efficiency under demand uncertainty is higher than under deterministic demand. As discussed in the previous section, firm 2, the low-quality firm, only produces if firm 1 is at its capacity. This results in a lower capacity share for firm 2 than in the deterministic case. Since for equal capacity costs it would be welfare optimising to have firm 1 produce all output, a smaller capacity share for firm 2 is more efficient, hence the stochastic case is more efficient than the deterministic one. The difference in efficiency decreases as the quality difference declines (i.e. θ closer to unity). Moreover, as we have seen earlier, total capacity is higher under uncertainty at low-cost levels, partly compensating the welfare loss due to duopoly behaviour.⁸ Under deterministic demand, efficiency first decreases in the degree of vertical differentiation, but increases for high degrees of differentiation. The reason for this pattern is that, starting from high degrees of differentiation, a decrease in differentiation implies a

⁸ The latter effect is smaller for higher (but still equal) cost levels, so that the difference in efficiency between the stochastic and deterministic case decreases.

larger, inefficient, capacity share for the low-quality firm 2. However, if the degree of differentiation decreases further, the efficiency increases because the quality of firm 2 increases and hence the efficiency loss caused by firm 2 producing instead of firm 1 decreases.

The picture is somewhat different if costs per unit of capacity are unequal, i.e. if the (perceived) high-quality firm has higher costs, as shown in the right panel of Figure 4. The efficiency results for different costs per firm follow from the profit maximising capacity shares of the firms (Figure 2) versus their welfare maximising capacity shares (Figure 3). Since both firms will always have positive capacities, the highest level of efficiency is reached at the degree of horizontal product differentiation where it would be optimal for welfare if firms had equal capacities. Both under deterministic and stochastic demand, profit maximising outcomes yield capacities for the least efficient firm (i.e. high cost relative to quality) that are too high from a welfare point of view. The stronger peak for the efficiency curve for deterministic demand in the right panel corresponds to the sharp switch in welfare optimizing capacities as shown in Figure 3.

5.4 Price dispersion

Our model provides some insights in the ongoing debate on the impact of the level of competition on price dispersion. Dana (1999) sets up a model using fixed capacity under demand uncertainty and finds that intrafirm price dispersion increases in the number of firms in the market, i.e. the more competitive a market is, the larger the dispersion in prices. Dana relates his theoretical finding to earlier empirical results in civil aviation (Borenstein and Rose, 1994). Several other empirical studies on intrafirm price dispersion in civil aviation find mixed results however. Like Borenstein and Rose (1994), Hayes and Ross (1998), and Giaume and Guillou (2004) find a positive relationship between the level of competition and price dispersion, whereas Gerardi and Shapiro (2009) find a negative relationship. Dai et al. (2012) find

the relationship to be parabolic. Orlov (2011) finds an indirect positive relationship through lower search costs.⁹

The numerical outcomes from our model provide an explanation for the ambiguity in empirical results. We compare levels of intrafirm price dispersion (over demand states) for different degrees of vertical product differentiation. Vertical product differentiation is negatively related to the competitiveness of the market, as more homogeneous goods lead to fiercer competition.¹⁰ Our results show that price dispersion decreases in the level of vertical product differentiation, hence increases in competition, for the high-quality firm, whereas this relationship is parabolic for the low-quality firm.

6 Conclusion

We model capacity-then-price competition under demand uncertainty in a duopoly with product differentiation. We conclude that a subgame perfect Nash equilibrium (SPNE) in pure strategies only exists if the market is characterised by a sufficient degree of vertical product differentiation relative to horizontal differentiation. This sufficient degree allows us to solve the pricing game, via backward induction, by defining three different regions – contested monopoly, residual monopoly and Bertrand Edgeworth duopoly – based on actual level of demand and installed capacities. To the best of our knowledge, this is the first study to find equilibria for capacity-then-price games addressing explicitly

⁹ Mantin and Koo (2009) find no relationship between competition and price dispersion, but it has to be noted that they analyse price dispersion on the route level, whereas the other studies analyse intrafirm price dispersion only.

¹⁰ Moreover, more homogenous goods yield more symmetric outcomes, and is hence related to lower market concentration levels, which generally used as a (negative) proxy for competitiveness in empirical studies. It can be checked from Figure 2 that the negative relationship between market concentration and θ no longer holds if cost differences are introduced.

both demand uncertainty and product differentiation, without having to rely on mixed strategies and/or submodular games.

Apart from establishing the existence of the SPNE in pure strategies, this article looks at the effects of per unit capacity costs, demand uncertainty and the degree of vertical and horizontal product differentiation on installed equilibrium capacities, welfare efficiency, and price dispersion. We find that given the required degree of vertical product differentiation to guarantee the SPNE in pure strategies, horizontal product differentiation hardly has an effect on the outcome of the model in terms of prices, capacities and efficiency.

In case of low capacity costs, both profit- and welfare maximising capacities are higher under stochastic demand compared to deterministic demand. As these costs increase, having spare capacity for low levels of demand becomes too expensive compared to the forgone profit and welfare due to binding capacity constraints for high levels of demand. Consequently, for high capacity costs profit- and welfare maximising capacities are lower under stochastic demand compared to deterministic demand. For asymmetric capacity costs, the results show that under stochastic demand the model yields asymmetric outcomes for parameter values – costs and product differentiation levels – that would result in symmetric outcomes for deterministic demand. This finding is consistent with the results by Reynolds and Wilson (2000), and De Frutos and Fabra (2011). The highest level of efficiency is reached at the degree of vertical product differentiation where it would be optimal for welfare if firms had equal capacities.

We apply our model to study price dispersion. The findings indicate that vertical product differentiation affects price dispersion of the high-quality firm negatively, whereas for the low-quality firm a positive

relationship exists. This opposite direction between firms may explain the ambiguous results found in empirical studies.

This article identifies three main directions for further research. The first direction can be found in generalising our model for larger numbers of competitors. Benassy (1989) establishes that equilibrium existence is ensured for horizontal product differentiation if the number of competitors is sufficiently large. This implies that our condition on the degree of vertical product differentiation could probably be relaxed for larger numbers of competitors. Incorporating this into our model can provide a more explicit relationship between the condition and the number of players in the market. The second direction lies in testing the model empirically, which would shed light on the existence issue as well as on the results found here. Key in these kind of empirical analyses is to measure the degree of product differentiation adequately. Finally, this article assumes the degree of product differentiation to be given, whereas one could argue that firms in reality may choose, for example, their quality and location, and therefore the degree of product differentiation should be modelled endogenously.

Appendix A: Degree of vertical product differentiation and the low-quality firm

Using the minimum demand firm 2 faces, this appendix shows that firm 2 does not produce at all if the degree of vertical differentiation is too high, i.e. if θ does not exceed $\phi/(2-\phi^2)$. The minimum level of demand occurs if both firms play the ordinary Bertrand pricing game. Rewriting inverse demand, as specified in Eq. (2), yields the relevant demand function for each output:

$$\begin{cases} D_{1,B}(p_1, p_2) = ((\alpha(1-\theta\phi) - p_1 + \phi p_2)b)/(1-\phi^2), \\ D_{2,B}(p_1, p_2) = ((\alpha(\theta-\phi) - p_2 + \phi p_1)b)/(1-\phi^2). \end{cases} \quad (\text{A.1})$$

The profit function simply equals demand multiplied by the corresponding price. Taking the first order conditions with respect to prices and solving for both prices gives the equilibrium price for both outputs:

$$\begin{cases} p_{1,B}^* = \alpha(2-\phi^2-\phi\theta)/(4-\phi^2), \\ p_{2,B}^* = \alpha(2\theta-\phi^2\theta-\phi)/(4-\phi^2). \end{cases} \quad (\text{A.2})$$

Equilibrium outputs are determined by substituting these prices into the demand function Eq. (A.1):

$$\begin{cases} z_{1,B}^* = \alpha b(2-\phi^2-\phi\theta)/((4-\phi^2)(1-\phi^2)), \\ z_{2,B}^* = \alpha b(2\theta-\phi^2\theta-\phi)/((4-\phi^2)(1-\phi^2)). \end{cases} \quad (\text{A.3})$$

It follows directly from Eq. (A.2) and Eq. (A.3) that a necessary condition for both $p_{2,B}^*$ and $z_{2,B}^*$ to be positive is $\theta > \phi/(2-\phi^2)$. As a consequence, firm 2 is not active if $\theta \leq \phi/(2-\phi^2)$, i.e. if the quality difference between the two goods as perceived by the consumers is too large.

Appendix B: Contested or residual monopoly

This appendix determines the degree of vertical product differentiation needed for which firm 1 has the incentive to saturate firm 2's capacity. Firm 1's profits as a contestable monopolist are easily determined by multiplying the equilibrium price (Eq. (6)) and output (Eq. (7)): $\pi_{1,CM}^* = \alpha^2 b \theta (\phi - \theta) / \phi^2$.

To calculate the profits if firm 1 chooses to play as a residual monopolist, we first consider firm 2's best response function in prices: $p_{2,RM_1} = (\alpha(\theta - \phi) + \phi p_{1,RM_1}) / 2$. Rewriting this function for p_{1,RM_1} and substituting the clearing equilibrium price p_{2,RM_1}^* for which $D_2(p_{2,RM_1}^*) = x_2$, yields the equilibrium price for firm 1: $p_{1,RM_1}^* = (\alpha(\phi - \theta) + 2p_{2,RM_1}^*) / \phi = (\alpha b(\phi - \theta) + 2x_2(1 - \phi^2)) / b\phi$. Substituting this price into the demand function yields equilibrium output: $z_{1,RM_1}^* = (\alpha b \theta - x_2(2 - \phi^2)) / \phi$. Therefore the profit as a residual monopolist equals:

$$\pi_{1,RM_1}^* = (\alpha b \theta - x_2(2 - \phi^2)) (\alpha b(\phi - \theta) + 2x_2(1 - \phi^2)) / b\phi^2. \quad (B.1)$$

Because we are interested for which degree of vertical product differentiation it is more profitable for firm 1 to act as residual monopolist, we solve $\pi_{1,CM}^* > \pi_{1,RM_1}^*$ for θ :

$$\theta \leq (2 - \phi^2) (\alpha b \phi + 2x_2(1 - \phi^2)) / \alpha b(4 - 3\phi^2). \quad (B.2)$$

Hence, if θ is smaller than the expression in Eq. (B.2), firm 1 does not have the incentive to act as residual monopolist. This condition is always satisfied for $\theta \leq \phi / (2 - \phi^2)$:

$$\phi / (2 - \phi^2) - (2 - \phi^2) (\alpha b \phi + 2x_2 (1 - \phi^2)) / \alpha b (4 - 3\phi^2) = \frac{(1 - \phi^2) (\alpha b \phi^3 - 2(2 - \phi^2)^2)}{\alpha b (2 - \phi^2) (4 - 3\phi^2)}, \quad (\text{B.3})$$

which is necessarily negative since $\alpha b \phi^3 < 2(2 - \phi^2)^2$ for any level of demand b and for $0 \leq \phi \leq 1$.

Appendix C: Pure or contested monopoly

Suppose firm 1 acts as a monopolist rather than a contested monopolist. Monopoly profits are equal to:

$\pi_{1,M}^* = (1/4)\alpha^2 b$, whereas contested monopoly profits equal: $\pi_{1,CM}^* = (\alpha^2 b \theta (\phi - \theta)) / \phi^2$. For $\pi_{1,CM}^* > \pi_{1,M}^*$

it is required that $\theta > (1/2)\phi$. It is only profitable for firm 1 to act as a monopolist if firm 2 cannot

produce profitably at the monopoly price. Substituting this monopoly price, $p_{1,M}^* = (1/2)\alpha$, into the best

response function of firm 2 yields firm 2's equilibrium price: $p_{2,M}^* = \alpha(2\theta - \phi)/4$. Obviously, $\theta \leq (1/2)\phi$

yields a zero or negative equilibrium price for firm 2.

Appendix D First-best solution

This appendix defines the first-best output levels in each region if $\theta < \phi / (2 - \phi^2)$. First, we assume equal capacity costs per firm and, consequently, that the output of firm 1 is superior. Therefore, it is efficient to have only firm 1 producing for all levels of demand. Substituting $z_2 = 0$ into the utility function and subtracting the costs of capacity yields the welfare function:

$$W_{fb} = \alpha z_1 - (z_1^2 / 2b) - c_1 x_1. \quad (D.1)$$

It can be readily verified that $z_{1,fb}^* = \alpha b$ and that the capacity of firm 1 is restricted from $b > x_1 / \alpha$ onwards. Therefore the optimal welfare as function of capacity becomes:¹¹

$$W_{fb} = \left(\frac{x_1}{\alpha} \right) \left(\alpha \left(\frac{x_1}{2} - \frac{\left(\frac{x_1}{2} \right)^2}{2 \frac{x_1}{2\alpha}} \right) - \left(1 - \frac{x_1}{\alpha} \right) \left(\alpha x_1 - \frac{x_1^2}{2 \left(\frac{1}{2} + \frac{x_1}{2\alpha} \right)} \right) - c_{x,1} x_1 \right) = \frac{x_1^2}{4} + \frac{\alpha x_1 (\alpha - x_1)}{\alpha + x_1} - c_{x,1} x_1. \quad (D.2)$$

In the more realistic case that firm 1 has higher costs per unit of capacity, production of firm 2 increases welfare if firm 1 is at its capacity. Therefore, the first region is the same as in case of equal capacity costs, but there is an intermediate region for which firm 2 also produces. The welfare function in case firm 1 is at its capacity equals:

$$W_{fb} = \alpha (x_1 + \theta z_2) - (x_1^2 + z_2^2 + 2\phi x_1 z_2) / 2b - c_1 x_1 - c_2 x_2. \quad (D.3)$$

¹¹ Applying the uniform distribution for b as used throughout this article.

Optimising Eq. (D.3) with respect to z_2 yields the social optimal output: $z_{2,fb}^* = \alpha b \theta - \phi x_1$. This intermediate region is bounded by the level of demand, b , for which $z_{2,fb}^* = x_2 : b < (\phi x_1 + x_2) / \alpha \theta$.

The optimal outputs are $z_{1,fb}^* = \alpha b$, $z_{1,fb}^* = x_1$, $z_{1,fb}^* = x_1$, and $z_{2,fb}^* = 0$, $z_{2,fb}^* = \alpha b \theta - \phi x_1$, $z_{2,fb}^* = x_2$ for region 1, and 2, and 3 respectively. Furthermore, the boundary conditions are $b < x_1 / \alpha$ and $b < (\phi x_1 + x_2) / \alpha \theta$ for region 1 and 2. Combining the above with the uniform distribution of the level of demand, which simplifies the applicable b for each region to the average of the lower and higher boundary value of b , the welfare function yields:

$$W_{fb} = x_1^2 / 4 + \Theta (x_1 (\phi - \theta) + x_2) / \alpha \theta + (1 - (\phi x_1 + x_2) / \alpha \theta) \Omega - c_{x,1} x_1 - c_{x,2} x_2, \quad (D.4)$$

with $\Theta = \alpha (x_1^2 (4\phi + \theta^3 + \theta\phi^2 - 2\theta^2\phi) + x_1 x_2 (2\theta(\theta - \phi) + 4) + \theta x_2^2) / 4 (x_1 (\theta + \phi) + x_2)$,

and $\Omega = \alpha (x_1 (x_1 (\phi - \theta) + (\alpha \theta + (1 - \theta \phi) x_2)) + x_2 \alpha \theta^2) / (\alpha \theta + x_1 \phi + x_2)$.

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